Math 10A with Professor Stankova
Quiz 5; Wednesday, 9/27/2017
Section #107; Time: 11 AM
GSI name: Roy Zhao **Name:** ______________________

Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. **TRUE** False It is possible for Newton's method to fail.
2. **TRUE** False You can change an exponential indeterminate into a form suitable for using L'Hopital's rule.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (5 points) Approximate $(1.1)^{0.1}$ using second order Taylor series. You may leave your answer as a sum of fractions.

Solution: We want to approximate the function $f(x) = x^{0.1} = x^{1/10}$. A good base point is a value nearby which is $1^{1/10} = 1$. So expanding $f(x)$ around $x = 1$ gives

$$f(x) \approx f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2 = 1 + \frac{x-1}{10} - \frac{9(x-1)^2}{200}.$$

Now we plug in $x = 1.1$ to get

$$(1.1)^{0.1} \approx 1 + \frac{0.1}{10} - \frac{9(0.1)^2}{200} = \boxed{1 + \frac{1}{100} - \frac{9}{2 \cdot 10^4}}.$$

(b) (1 point) When using Newton's method to find a zero of a function $f(x)$, what is the formula for the next guess x_1 if my current guess is x_0?

Solution:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

(c) (4 points) Use Newton's method once to approximate $(1.1)^{0.1}$.

Solution: Our function that we want to find a zero of is not $x^{0.1}$ but $x^{10} - 1.1$. Our initial guess is $x_0 = 1$. Now we use the above formula to get that our next guess is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{1 - 1.1}{10 \cdot 1^9} = 1 + \frac{0.1}{10} = \boxed{1.01}.$$